Section 5.6 Price Elasticity of Demand

Formula: $E = -\frac{dq}{dp} \cdot \frac{p}{q}$ lowercase

$E < 1$ Inelastic

where max occurs $E = 1$ unit elasticity

$E > 1$ Elastic

#6 pg. 437, where $q = \frac{(400-p)^2}{100}$ $0 \le p \le 400$

Previous equations: $y = x$, but now $q = p$; (y) becomes our (q) + (x) becomes (p)

form we use: $q = \ldots\ p$

Part a Find E when $p = \$40^{00}$

Step 1 Find $\frac{dq}{dp} =$, so find derivative. Algebraically readjust to make it easier

, so $\frac{1}{100}(400-p)^2$

Using Power Rule: $\frac{1}{100} \cdot 2(400-p)^1 \cdot (-1)$

Bring down power; new power b/c $2-1=1$; Inside derivative $(400-p)$

Step 2 $= \frac{-(400-p)}{50}$

Step 3 $E = -\frac{dq}{dp} \cdot \frac{p}{q}$ Plug in pieces to our equation, so

$$E = --\frac{(400-p)}{50} \cdot \frac{p}{\left(\frac{(400-p)^2}{100}\right)}$$

Step 4 Clean up algebra:

$$E = \frac{\cancel{(400-p)}}{\cancel{50}} \cdot \frac{p}{1} \cdot \frac{\cancel{100}\,2}{(400-p)^{\cancel{2}}} = \frac{2p}{400-p}$$

* Elasticity of D equation *

Part a – question: Find E when $p = \$40$, so plug in \$40 to our equation

$E = ?$ when $p = \$40$

$$E = \frac{2(40)}{400-40} = \frac{80}{360} = \boxed{0.\overline{22}}$$

B|C $E<1$ → Inelastic D, states as price increases our profit will not be negatively affected

Part b price for max revenue?

when $E = 1$

Step 1 Use equation we created:

$E = \frac{2p}{400-p}$ and set = to 1, so

$E = \frac{2p}{400-p} = 1$; $2p = 400 - p$; $3p = 400$

$p = \$133.33$ should change this much to maximize revenue

Part C What is the maximum weekly revenue?

Step 1, so $R = p * q$; use $q = \frac{(400-p)^2}{100}$

Step 2 $q = \frac{(400-133.33)^2}{100}$; $q = 711.13$

$\boxed{\text{Step 3}}$ Total Revenue $= p \cdot q = (133.33)(711)$

$$\boxed{= \$94{,}797.63}$$

#8 Looking for elasticity of demand when $p = 4$ GOAL!!

$\boxed{\text{Step 1}}$ Demand function $p = \dfrac{40}{q^{1.5}}$

ALWAYS Have to Δ to format $q = \ldots p$

$\boxed{\text{Step 2}}$ $p \cdot q^{1.5} = 40$

$\boxed{\text{Step 3}}$ $q^{1.5} = \dfrac{40}{p}$

$\boxed{\text{Step 4}}$ Undue power of $q^{1.5}$ by changing 1.5 to fraction $3/2$, then multiply both sides by $2/3$

$$\left(q^{3/2}\right)^{2/3} = \left(\frac{40}{p}\right)^{2/3} \qquad q = \frac{40^{2/3}}{p^{2/3}} \text{ or } q = 40^{2/3} \cdot p^{-2/3}$$

$\boxed{\text{Step 5}}$ $E = -\dfrac{dq}{dp} \cdot \dfrac{p}{q}$; so

$$\frac{dq}{dp} = 40^{2/3} \cdot -\frac{2}{3} p^{-2/3-1} = -\frac{2 \cdot 40^{2/3}}{3p^{5/3}}$$

$\boxed{\text{Step 6}}$ $-\left(\dfrac{-2 \cdot 40^{2/3}}{3 \cdot p^{5/3}}\right) \dfrac{p}{\left(\frac{40^{2/3}}{p^{2/3}}\right)}$

$\boxed{\text{Step 7}}$ Get rid of denominator

$$= \frac{2\cancel{(40^{2/3})}}{3\cancel{p^{5/3}}} \cdot \frac{\cancel{p}}{1} \cdot \frac{\cancel{p^{2/3}}}{\cancel{40^{2/3}}} = \frac{2}{3}$$

$\boxed{\text{Step 8}}$ Find $E = ?$ when $p = 4$

B|C no (p) left $\boxed{E = \frac{2}{3} \text{ always (constant)}}$

$\boxed{\text{Step 9}}$ Interpret the result !!

Regardless of what price is Charged, the profit will continue to increase.